A Guide to Smart Beta and Systematic Factor Portfolio Construction Methods

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Factor investing is a popular investment strategy adopted by many hedge funds and asset managers. Its applications span performance breakdown, hedging, asset selection, and portfolio allocation. In this paper, we aim to provide an overview of two applications of factor investing—Smart Beta and Systematic Factor Portfolio Construction.

Factors are “attributes” that drive returns of assets. You can theoretically predict the risk and return of a stock or other asset based on its relationship to a factor or a group of factors. Factor investing simply uses regression models to analyze this relationship, and the resulting value derived from a regression model provides insight into an asset's returns. In the regression model, the factor is simply the independent variable or regressor.

You can think of a factor as a portfolio that has its own return and variation statistics, very much like that of a stock or investment portfolio. In fact, factors are oftentimes constructed as a portfolio of different stocks using different asset allocation parameters.

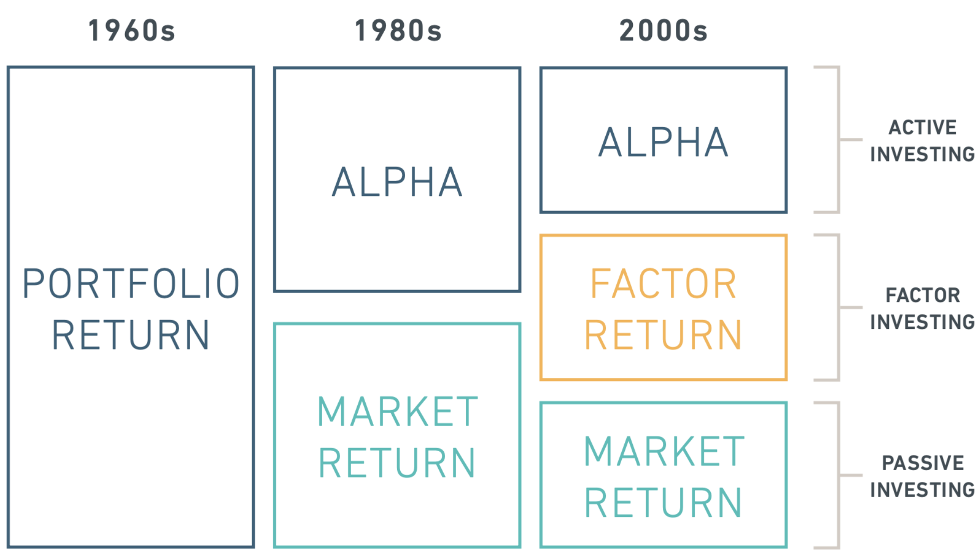


Figure 1: Construction of a twice-tiled factor index (Quality + Value Factors)
Source: MSCI

Types of Factors

Factors can generally be broken down into style and macroeconomic factors. Style factors relate to the characteristics of an asset or a company. Some popular style factors include the value factor, momentum factor, yield factor, and size factor. Macroeconomic factors relate to the effects of macroeconomic forces on the returns of an asset. This includes interest rates, inflation, GDP growth, and any other relevant macroeconomic influences on the economy.

Different funds categorize their factors differently. For example, the MSCI's Barra Global Equity Factor Model categorizes 16 factors into 8 different factor groups (Figure 10), illustrating how factors could be derived from measurable characteristics.

VALUE	SIZE	MOMENTUM	QUALITY
Book-to-Price **Earnings Yield** **Long-Term Reversal**	**Size** **Mid Cap**	**Momentum**	**Leverage** **Earnings Variability** **Earnings Quality** **Investment Quality** **Profitability**
YIELD	VOLATILITY	GROWTH	LIQUIDITY
Dividend Yield	**Beta** **Residual Volatility**	**Growth**	**Liquidity**

Figure 2: MSCI Factor Categorization

Source: MSCI

Part I: Factor Construction Models

Multi-Factor Portfolio Construction Decisions

Asset managers are confronted with a set of decisions influencing their approach to constructing multi-factor portfolios. Some of the choices available could be summarized in the table as follows:

Portfolio Construction Variable	Details	Consideration
Top-Down vs. Bottom-Up	*Top-Down*: Capture exposure to multiple factors by combining multiple single-factor indices *Bottom Up*: Capture exposure to multiple factors by selecting securities that score highly across all factors on average	A Top-Down approach would provide more diversification but with a greater chance of factor dilution. A Bottom-Up approach would provide higher factor exposures.
Sector-Neutral vs. Sector-Agnostic	*Sector-Neutral*: Stock selection is conducted independently within each sector to hit a sector weight target *Sector-Agnostic*: Stock selection is conducted purely by factor score without any sector constraint	Sector-Neutral strategies would reduce exposure to unintended risks, but Sector-Agnostic strategies would
Rebalancing Frequency	Reselect and reweight constituent securities on a regular basis	Trade-off between turnover costs and factor decay (Turnover costs are incurred every time a portfolio manager trades securities; Factor decay occurs when factor exposure decreases over time)
Factor Combinations	The type and combination of factors	Investors can choose to combine different factors (such as value and momentum) and weigh them accordingly

Source: S&P Global, *Exploring Techniques in Multi-factor Index Construction*

These considerations, among others, influence a portfolio manager's choice of multi-factor portfolio strategy. In the following sections, we will delve into a few multi-factor construction models used for security selection, followed by some portfolio weighting schemes employed in smart beta index construction.

Construction Models for Security Selection

Heuristic Construction Parameters

According to NASDAQ's *Practitioner's Guide to Multi-Factor Portfolio Construction*, a Heuristic Multi-factor approach could be used to derive a comprehensive factor score for a particular security. A factor score is simply an assigned score based on an assets relationship to a factor or factors. The process for using a heuristic construction model may follow the below process:

1. Identify the proper factor or factors you want to use in your model.
2. Run a factor regression analysis of every publicly traded company onto your set of factors.
3. For each company, calculate a factor score (there are many different ways to calculate this factor score.
4. Select the top n assets based on their factor scores and construct a portfolio based on these results.

By deriving a comprehensive factor score for each security within a selected universe of securities, the top *x* securities could be selected and fed into a portfolio weighting scheme such as mean-variance to get the final portfolio allocation weights and develop a systematically adjusted portfolio of assets..

The advantage of a heuristic weighting approach is that it is the most simple to implement. You just have to run a regression and calculate a "score" based on basic calculations from the result of the regression. However, this also means that the heuristic approach will have higher tracking errors, may not generate alpha efficiently, and will have a worse risk adjusted performance–based on tracking error and alpha–than other methods of factor construction

One such factor score can be calculated using the below formula.

$$\alpha_i = 0.2 * F_{1,i} + 0.2 * F_{2,i} + 0.2 * F_{3,i} + 0.2 * F_{4,i} + 0.2 * F_{5,i}$$

where α_i represents the comprehensive factor score for a security i, while $F_{i,j}$ represents the factor score of security i for a factor j.

Another example of constructing the factor score is using the following method based on the information ratio and normalizing the regression data. This model is based on raw factor scores being normalized such that their distribution has a mean of 0 and a standard deviation of 1. You do this by calculating the factor Z-scores. The Z-Score is obtained by the following:

$$Z = (x - \mu)/\sigma$$

where x, μ and σ represent the raw factor score, mean factor score, and standard deviation respectively

Here is an overview of how a factor Z-score could be obtained for a value factor:

A factor Z-score for the value factor could be based off of a security's Earnings Per Share (EPS). Here, the author adopts a 5-step process:

Step 1: Obtain the S&P 500 Index Composition over time

Step 2: Process Factor Data
Data sets undergo winsorization and industry neutralization to remove outlying scenarios and remove differences across industries respectively. After which, data points undergo a standardization process which is illustrated in Figure 3 below:

-	Stock 1	Stock 2
EPS	1.2	15.3
Industry	Finance	Consumer Goods
Industry EPS Standard	0.8	12.2
Difference	+0.4	+3.1
Industry EPS Standard Deviation	0.2	2
Standardized Score	0.4/0.2 = +2	3.½ = +1.5

Figure 3: Standardized score of 2 stocks

Step 3: Calculate the IC and IR to determine the correlation between the factor and next day's returns
Information Coefficient (IC): Correlation between the factors and next day's return. The higher the IC, the better the factor's ability to predict next day's return.

Information Ratio (IR): indicator of the stability of an IC. The higher the IR, the better an IC's stability; an acceptable range would be between 0.4 and 0.6; an IR of 1.0 is highly desirable.

The IR can be calculated using the following:

$$IR = \frac{IC}{standard\ deviation\ of\ IC}$$

Step 4: Calculate the factor scores
The final factor score for each security could be calculated using the following:

$$Factor\ score = \sum_{n=1}^{no.\ of\ factors} factor_n \times IR\ weight_n$$

Step 5: Verify the results

Factor scores have to be screened for their effectiveness, and this can be done by sorting the securities in order of their respective factor scores, and splitting them into N different groups. After which, chart the summation of the daily returns of each group and compare the findings across the different groups over time. Figure 4 illustrates the ideal outcome, while figure y illustrates a poor factor set. In an ideal outcome (Figure 4), lines on the graph rarely – if ever – cross over. Figure 5 illustrates a less-than-ideal outcome.

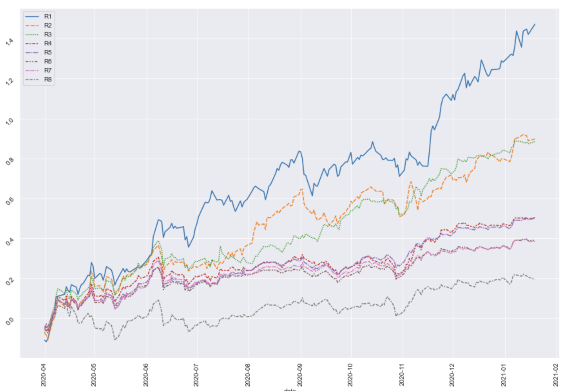


Figure 4: Ideal Outcome

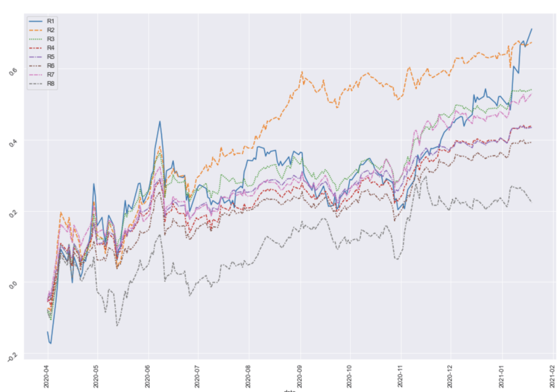


Figure 5: Poor Outcome

Source: Hsia, M. *[Factor Analysis] Vol.4. Use factor score to quantify the growth tendency of stock return.*

<u>Optimized Multi-Factor Construction</u>

In constructing multi-factor portfolios, investors can control the active risk of a portfolio, or the risk characteristics of an actively managed portfolio relative to a benchmark, through the introduction of a risk optimization framework.

One approach is active risk, or tracking error minimization. Tracking error, which is a measure of the divergence between the return profile of a portfolio and its corresponding benchmark, could be calculated as follows:

$$Tracking\ Error\ =\ \sqrt{var(r_p - r_b)}$$

where r_p *and* r_b *represent the return of a portfolio and the return of a benchmark respectively.*

Tracking error is thus indicative of a portfolio's performance divergence from the benchmark; a positive tracking error suggests outperformance while a negative one, lagging performance. Traditional passive fund managers typically aim to minimize tracking error, resulting in fund performance that largely mirrors that of the benchmark.

While there are many ways in which the tracking error minimization problem could be solved, a challenge faced by portfolio managers is the minimization of tracking error for a portfolio of a small number of stocks. As such, this section will be focused on a graduated non-convexity approach to the cardinality-constrained tracking error minimization problem that introduces a constraint on the total number of assets, N. Here, a cardinality constraint is one that sets a limit on the number of elements, and in this case, stocks, within a set or portfolio.

We will adopt the example authored by Henniger, Li and Coleman (2006), which seeks to choose a portfolio of 25 stocks tracking the S&P500 index given a preallocated portfolio of *N* stocks, such that $1 \leq N \leq 500$ and holdings of the preallocated portfolio could be found in the S&P500.

Let w_i be the weighting of stock i in the portfolio, such that $1 \leq i \leq N$, with N being an arbitrary constant. Three key variables are defined as follows:

1. *w* represents a vector of weightings of stocks in the preallocated portfolio;
2. *x* represents a vector of the index weightings of the same set of stocks (S&P500);
3. Q represents the positive definite covariance matrix of stock returns.

We suppose the tracking error function to be the following:

$$TE(w) = (w - x)'\, Q\, (w - x)$$

where $(w - x)'$ *is the transpose of* $(w - x)$

This is a convex function (see below for explanation) and is desirable both in terms of its mathematical characteristics and interpretability relative to other tracking error functions. A tracking error of 1% implies an expected tracking portfolio return within $\pm$ 1% with a 67% probability and a tracking error of 2% implies an expected tracking portfolio return within $\pm$ 2% with a 95% probability. Notice that tracking error is least when $w = x$.

In a basic heuristic algorithm, the optimal portfolio of 25 stocks could be selected by solving a quadratic programming problem described by Jansen and van Dijk (2002). The first step is to find the best weights, x_i, in the portfolio that minimizes the tracking error. The initial optimal tracking error would be such that $w = x$. In this solution, we remove *n* stocks with the smallest weights and proceed with another iteration of the problem using the remaining number of stocks, $N - n$. This process goes on until we are left with 25 stocks. Note that n could represent any number of stocks. While this is a valid example of optimization, we want to find another way of achieving a similar outcome without having to solve a potentially prohibitive number of index tracking sub-problems.

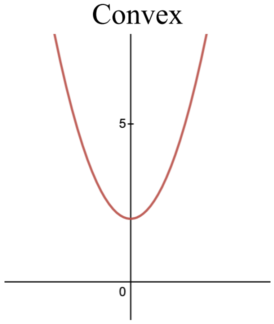
The underlying concept behind the tracking error minimization problem subject to a cardinality constraint is the computation of a global minimum of an objective function. This objective function involves a function computing the tracking error (see above) as well as a discontinuous counting function $\sum_{i=1}^{N} \Lambda(w_i)$, where $\Lambda(w_i) = 1$ if $w_i \neq 0$, and 0 otherwise.

In order to overcome various mathematical difficulties of the tracking error minimization problem such as the lack of differentiability of approximations to the discontinuous function, the authors proposed a solution that approximates the discontinuous function $\Lambda(w_i)$ by a sequence of "*continuously differentiable non-convex piecewise quadratic functions*" approaching $\Lambda(w_i)$ in the limit.

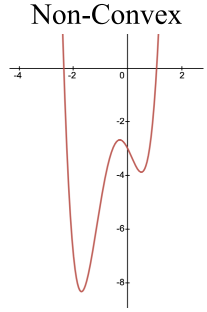
Here, we will provide a brief explanation of the concepts *global minimum*, *local minimum*, *non-convex*, and *piecewise*.

Below illustrates 3 different types of functions. A convex function has one minimum point, a non-convex function has multiple minimum points, while a concave function has one maximum point. A global minimum is essentially the smallest possible minimum point, while a local minimum can be any minimum point on a graph. As such, a convex function has a local minimum that is equal to the global minimum. This is an important consideration in optimization problems because oftentimes, one might yield to a solution that is a local minimum and not the global minimum.

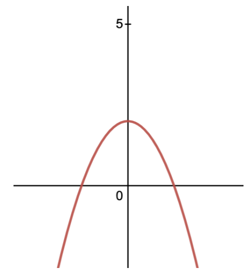
Convex



Non-Convex



Concave



Piecewise functions, on the other hand, are simply functions composed of multiple functions within fixed boundaries. For instance, we can have a function $f(x)$ such that $f(x) = x$ for $0 \geq x \geq 1$ and $f(x) = x^2$ for $1 > x \geq 2$. A piecewise continuously differentiable function could sometimes be referred to as a piecewise smooth function.

On a high level, the tracking error minimization problem subject to a constraint on the total number of assets, K can be expressed using the following. This is an example of a *discontinuous optimization problem* where $\sum_{i=1}^{N} \Lambda(w_i) \leq K$ is the cardinality constraint function.

$$min\, TE(w),\ where\ w \in R^n$$
$$subject\ to\ \sum_{i=1}^{N} \Lambda(w_i) \leq K;\ \sum_{i=1}^{N} w_i = 1;\ w \geq 0$$

For the purpose of simplicity, we adopt the following expression that is of equivalence to the one above:

$$min\, (TE(w)\ +\ \mu \sum_{i=1}^{N} \Lambda(w_i)),\ where\ w \in R^n\ and\ \mu \geq 0$$
$$subject\ to\ \sum_{i=1}^{N} w_i = 1;\ w \geq 0$$

μ is a penalty parameter. The penalty function $\mu \sum_{i=1}^{N} \Lambda(w_i)$ effectively applies a constraint similar to $\sum_{i=1}^{N} \Lambda(w_i) \leq K$. By varying μ, the solution would be able to yield optimal portfolios with different desired numbers of assets, K.

The information presented thus far is the basis for understanding the 3-step solution of the tracking error minimization problem.

Let λ be a large constant such that $\lambda > 0$ and $\{\rho_k\}$ be a monotonically increasing sequence that converges to +∞.

Step 1: We first minimize the tracking error without the cardinality constraint by solving the problem shown below:

$$min\, TE(w),\ where\ w \in R^n$$
$$subject\ to\ \sum_{i=1}^{N} w_i = 1; w \geq 0$$

Given the convexity of the tracking function, if we have a solution $w\ *$ that also satisfies $\sum_{i=1}^{N} \Lambda(w_i\, *) \leq K$, then the proposed method is guaranteed to yield solution $w\ *$ that satisfies the cardinality constraint because the local minimum is the global minimum.

Step 2: Compute a solution to the following problem, P_k, using the solution of the approximation P_{k-1} as a starting point. The following equations describe the approximation problem, P_k:

$$min\, (TE(w)\ +\ \mu max(\sum_{i=1}^{N} g_\lambda(w_i;\ \rho_k)\ -\ K,\ 0)),\ where\ w \in R^n$$

$$subject\ to\ \sum_{i=1}^{N} w_i = 1; w \geq 0$$

Brief derivation of $min\,(TE(w)\ +\ \mu max(\sum_{i=1}^{N} g\lambda(w_i;\ \rho_k) - K,\ 0))$

Recall from above the introduction of the penalty parameter, μ, resulting in the following equation: $min\,(TE(w)\ +\ \mu \sum_{i=1}^{N} \Lambda(w_i))$.

We want to find a series of approximations $\{P_k\}_{k=1,2,...}$. We first begin by finding a solution that minimizes the tracking error globally without non-convexity from the cardinality constraint, which is effectively what step 1 aims to achieve. This gives us solution P_1, which is used as starting point for the approximation problem P_2. Here, we gradually introduce non-convexity to incorporate the cardinality constraint, such that solution P_{k-1} is used as starting point for the approximation problem P_k.

To achieve this, we will need to approximate the counting function $\Lambda(w_i)$ with continuously differentiable piecewise quadratic functions with graduated non-convexity, which is achieved via two steps outlined below.

(i) We first approximate the discontinuous counting function $\Lambda(w_i)$ using a continuous function $h_\lambda(w_i)$:

$$h_\lambda(w_i) = \lambda z^2\ if\ |w_i| \leq \sqrt{\tfrac{1}{\lambda}};\ h_\lambda(w_i) = 1\ otherwise$$

$\lambda > 0$ is a large constant and is used in image reconstruction by Blake and Zisserman (1987), the details of which shall not be elaborated here. This gives us a continuous but non-differentiable problem given by the following:

$$min\,(TE(w)\ +\ \mu \sum_{i=1}^{N} h_\lambda(w_i)),\ where\ w \in R^n\ and\ \mu \geq 0$$

$$subject\ to\ \sum_{i=1}^{N} w_i = 1;\ w \geq 0$$

(ii) In this second step, we approximate the continuous, non-differentiable function $h_\lambda(w_i)$ using $g_\lambda(w_i; \rho)$:

$$g_\lambda(w_i; \rho) = \lambda w_i^2\ if\ |w_i| \leq q;$$

$$g_\lambda(w_i; \rho) = 1 - \tfrac{\rho}{2}(|w_i| - r)^2 if\ q \leq |w_i| < r;$$

$$g_\lambda(w_i; \rho) = 1\ otherwise$$

Note that q and r are 2 bounds such that $r = \sqrt{\frac{2}{\rho} + \frac{1}{\lambda}}$ and $q = \frac{1}{\lambda r}$

Given $\{\rho_k\}$, a monotonically increasing sequence introduced above, we find that as ρ_k increases, curvature of $g_\lambda(w_i; \rho)$ becomes more negative, which introduces a *graduated nonconvexity*.

As such, we now find that the discontinuous counting function $\Lambda(w_i)$ could be approximated using a continuous, differentiable function $g_\lambda(w_i; \rho)$ as shown below:

$$min\,(TE(w) \ + \ \mu \sum_{i=1}^{N} g_\lambda(w_i; \rho))$$

Finally, recall from above that adjustments to the penalty parameter μ would yield different values of K. Because a portfolio manager wants to obtain a portfolio with an upper bound K as input, the optimization problem could be adjusted for a more direct way of obtaining the desired solution:

$$min\,(TE(w) \ + \ \mu\, max(\sum_{i=1}^{N} g_\lambda(w_i; \rho_k) \ - \ K, 0))$$

Note that we use the max(a, b) where $a, b \in R$ to ensure that output values are greater than 0.

Step 3: Terminate the iteration if $(x_i)_k \le q_k$ or $(x_i)_k \le r_k$. Otherwise, we return to step 1 to find the solution of the approximation at P_{k+1}.

Solving this tracking error minimization problem would effectively yield a tracking portfolio of desired size.

Principal Component Analysis (PCA)

The key idea underlying a principle component analysis (PCA) is that five-factor models are not perfect ways to explain risk (Katchova, A., 2013). The PCA thus derives a set of empirically-driven and uncorrelated factors with no predefined factor scores that achieve maximum variance.

Mathematically, a PCA serves to find a set of factors $z = [z_1, z_2, z_3, \dots z_n]$, which are a set of linear combinations $u = [u_1, u_2, u_3, \dots u_n]$ of original variables $x = [x_1, x_2, x_3, \dots x_n]$ such that maximum variance is obtained.

1. The first factor, z_1, explains the maximum possible variance while the succeeding component, z_2, (that is uncorrelated to the first factor) explains any maximum possible variance not captured by the first. Each succeeding factor (z_3, z_4, etc.) then adopts a similar relationship to its preceding factor.
2. An eigenvalue decomposition of a sample correlation matrix, R, would allow one to obtain the solution, which is a set of eigenvectors as well as their accompanying eigenvalues.
3. The eigenvalues represent the importance of the component in explaining risk. Generally, an eigenvalue of less than 1 is considered to be less significant. Figure 6 (below) illustrates the retention of factors whose Eigenvalues are above 1.
4. Finally, the factor loadings, F are the correlations between x and z, and are denoted by:
   $F = corr\,(x, z) = uD^{1/2}$, where D is the diagonal covariance matrix of the factors, z

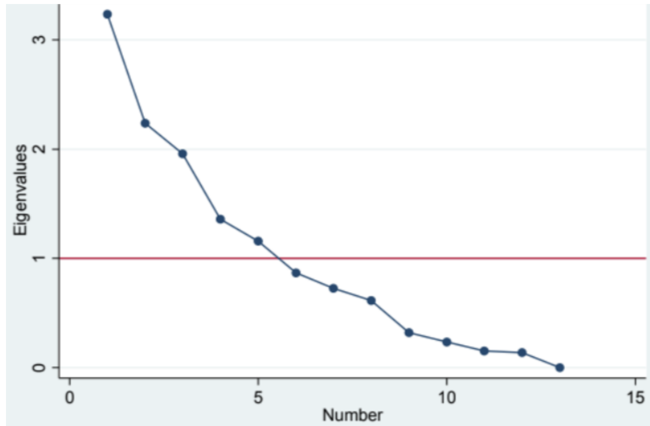


Figure 6: A plot of each succeeding factor's eigenvalue
Source: *Katchova, A., Principal Component Analysis and Factor Analysis.*

	ANZ	WBC	CBA	NAB
ANZ	1	0.97	0.96	0.85
WBC	0.97	1	0.98	0.76
CBA	0.96	0.98	1	0.73
NAB	0.85	0.76	0.73	1

Figure 7: Example of a correlation table between 4 securities
Source: Yang, Libin., *An Application of Principal Component Analysis to Stock Portfolio Management.*

When PCA is applied to a set of *n* securities, we can expect *n* uncorrelated sources of risk inherent to the original set of data. As above, the eigenvalues of each succeeding principal component decreases exponentially and become less relevant sources of risk (Yang, Libin., 2015). The top few eigenvectors selected are major contributors to risk.

The PCA could also be applied on a list of factors to identify the most important factors contributing to a winning stock. Hargreaves and Mani's *The Selection of Winning Stocks Using Principal Component Analysis* adopted the PCA to narrow a list of 22 identified factors (using fundamental and financial data) to 2 key factors that explain 94.3% of the variance of total returns from an identified set of securities. Securities are then evaluated based on the 2 identified leading factors.

Part II: Smart Beta and Portfolio Weighting Schemes

Smart Betas are defined as "transparent, rules-based portfolios designed to provide exposure to specific factors, market segments, or systematic strategies" (Russell Investments, 2015). There are two fundamental types of smart beta – strategy-based and factor-based. Strategy-based smart betas place a weighting emphasis on company fundamentals such as sales, dividends or average five-year cash flows. Factor-based smart betas, on the other hand, base their selections on particular stock characteristics (such as volatility).

Here, we examine how a set of pre-selected securities could be weighted. In many of the illustrated indexes, securities are typically placed under an initial screen from a universe before their constituent weights are determined by a weighting scheme.

<u>Alternatively Weighted Indexes</u>

Alternatively weighted indexes as designed by FTSE Russell enable investors to "target specific investment objectives" such as risk reduction. In this section, a few types of weighting schemes will be demonstrated via an overview of some alternatively weighted indexes.

(i) Equally Weighted Index

An index where capital is allocated equally across all constituent securities.

(ii) Equal Risk Contribution (ERC) Index

The ERC Index weights all eligible securities by equalizing the risk contribution of each security (MSCI, 2017). The MSCI Europe ERC Index derives the ex-ante risk estimate from the GEM25 Barra Equity Model, where

$$RC_i = w_i * MCR_i$$

and RC_i , w_i and MCR_i represent the Risk Contribution, weight, and Marginal Contribution to the Index Risk of a security i respectively

(iii) Minimum Variance Weighting Scheme

There are a few approaches to volatility reduction. The first alternative is a risk-weighted approach, where indexes are weighted according to the inverse of their historical volatilities, where a security with a higher volatility is assigned a lower weight. A second approach is the construction of a low volatility factor index that aims at capturing the factor premia of low volatility securities.

According to FTSE Russell (2017), the minimum variance index outperforms its counterparts and adopts an optimization approach that minimizes a portfolio's variance. The methodology for the FTSE Russell Global Minimum Variance Index Series could be outlined in the following steps:

Step 1: The equation that models the long-only portfolio variance is as follows:

$$\sigma^2 = W^T C W$$

where W is the vector of stock weights, C is the covariance matrix, and W^T is the transpose of the W matrix

Step 2: Set appropriate constraints, such as the industry and country weights

Step 3: Run the optimization, which is an iterative process that aims to arrive at a better outcome with each step

(iv) Risk Efficient Index

Risk efficient indexes obtain the optimal security weightings via maximizing the expected Sharpe Ratio, which requires a security's (i) expected return in excess of risk-free rate and (ii) covariance matrix of its expected returns.

In constructing a Risk Efficient Index, the constitution used to obtain the efficient weights is the same as that of a cap-weighted index (Amenc, Goltz & Martellini, 2010).

The efficient weights is then a set of weights that result in the highest Sharpe ratio, and may be obtained by the following:

$$w^* = \arg\max_w \frac{w'\mu}{\sqrt{w'\Sigma w}}$$

where μ represents a vector of expected returns in excess of the risk free rate, and Σ is the covariance matrix for returns of each of the constituent securities

The vector of expected returns are based on risk/return estimations, while the covariance matrix is estimated using the principal component analysis – which we will not elaborate on for the purpose of simplicity.

The optimization problem above then yields a solution modeled by the following (FTSE Russell, 2022):

$$w^* = m\, \Sigma^{-1} \mu$$

where m is a scalar that ensures all constituent weights sum up to 1

The solution above thus yields a set of optimal weights that determine the final weights of constituent securities in the FTSE EDHEC-Risk Efficient Index Series (EIC), which is a poignant example of a Risk Efficient Index.

(v) Fundamental Indexes

Securities in fundamentally weighted indexes are weighted according to a variety of fundamental properties such as book value, revenue or earnings. Securities are typically placed under an initial screen to weed out undesirable characteristics, followed by a weighting system to determine their constituent weights. For the purpose of illustration, we will examine WisdomTree U.S. Dividend Indexes' Methodology.

Firstly, companies are placed under an initial screening criteria, such as a minimum market capitalization of $100 million and other exclusions, including preferred stocks, exchange traded funds and derivative securities. A weighting formula then aims to magnify the effects dividends play in an index's total returns.

The weighting factor, which is the cash dividends to be paid, is determined by the reported annual dividend per share multiplied by the common shares outstanding. Each constituent security's weight is then determined by its share of contribution to the total dividend stream estimated to be paid in the succeeding year. Other adjustments are being made for securities with a share exceeding 12%.

Multi-Factor Composite Approach

A Multi-Factor Composite Index provides exposure to multiple factors via a simple composition of multiple single-factor indexes. For instance, a simple, two-factor composite index could consist of 50% value and 50% volatility. A constraint of adopting this approach, however, is that this average process potentially dilutes exposures to the factors of interest.

Below illustrates two types of hypothetical single-factor index construction using three different securities:

	Cap weight index	X Quality score	= Unadjusted weights	Final normalized Quality weights
Occidental Petroleum	33.6%	X 0.40	= 13.4%	52.4%
Ford Motor	33.3%	X 0.31	= 10.2%	40.0%
Barclays	33.1%	X 0.06	= 1.9%	7.6%
Total	**100.0%**		**25.5%**	**100.0%**

Figure 8: Construction of a quality index using an arbitrary quality score
Source: FTSE Russell. (2017). *Multi-factor indexes: The power of tilting*

	Cap weight index	X Value score	= Unadjusted weights	Final normalized Value weights
Occidental Petroleum	33.6%	X 0.13	= 4.6%	11.5%
Ford Motor	33.3%	X 1.00	= 33.2%	83.5%
Barclays	33.1%	X 0.06	= 2.0%	5.1%
Total	**100.0%**		**39.8%**	**100.0%**

Figure 9: Construction of a value index using an arbitrary value score
Source: FTSE Russell. (2017). *Multi-factor indexes: The power of tilting*

The creation of a composite two-factor (in this instance, quality & value) index using the normalized weights of its constituent factor indexes is illustrated below:

	(Quality weight + Value weight)/2	= Quality + Value composite index weight
Occidental Petroleum	(52.4% + 11.5%)/2	= 31.9%
Ford Motor	(40.0% + 83.5%)/2	= 61.7%
Barclays	(7.6% + 5.1%)/2	= 6.4%
Total	**100.0%**	**100.0%**

Figure 10: Construction of a two-factor composite index
Source: FTSE Russell. (2017). *Multi-factor indexes: The power of tilting*

Sequential Tilt Approach

"Factor tilting" is the sequential application of factor tilts that results in an index that is first tilted towards a factor, followed by another (FTSE Russell, 2017).

The factor weight of a security in a single-tilt portfolio could be obtained by the following formula:

$$\widehat{W}_i = \frac{S_i * W_i}{\sum_{j \in U} S_j * W_j}$$

for an underlying index universe U, with underlying index weights W_i and the standard cumulative normal distribution function of the cross-sectional Z-Scores for a given factor be represented by S_i

Figures 11 and 12 below illustrates the construction of a single-tilt factor index using the above formula:

	Cap weight index	X Quality score	= Unadjusted weights	Final normalized Quality weights
Occidental Petroleum	33.6%	X 0.40	= 13.4%	52.4%
Ford Motor	33.3%	X 0.31	= 10.2%	40.0%
Barclays	33.1%	X 0.06	= 1.9%	7.6%
Total	**100.0%**		**25.5%**	**100.0%**

Figure 11: Construction of a single-tilt factor index (Quality Factor)
Source: FTSE Russell. (2017). *Multi-factor indexes: The power of tilting*

	Cap weight index	X Value score	= Unadjusted weights	Final normalized Value weights
Occidental Petroleum	33.6%	X 0.13	= 4.6%	11.5%
Ford Motor	33.3%	X 1.00	= 33.2%	83.5%
Barclays	33.1%	X 0.06	= 2.0%	5.1%
Total	**100.0%**		**39.8%**	**100.0%**

Figure 12: Construction of a single-tilt factor index (Value Factor)
Source: FTSE Russell. (2017). *Multi-factor indexes: The power of tilting*

The factor weight of a portfolio in a twice-tilted index is thus given by:

$$\widehat{W}_i = \frac{S_{1i} * S_{2i} * W_i}{\sum_{j \in U} S_{1j} * S_{2j} * W_j}$$

where S_{1i} and S_{2i} represent the standard cumulative normal distribution function of the cross-sectional Z-Scores for given factors 1 and 2 respectively

	Cap weights	X Quality score	X Value score	= Unadjusted weights	Final normalized Value weights
Occidental Petroleum	33.6%	X 0.40	X 0.13	= 1.8%	15.0%
Ford Motor	33.3%	X 0.31	X 1.00	= 10.2%	84.0%
Barclays	33.1%	X 0.06	X 0.06	= 0.1%	1.0%
Total	**100.0%**			**12.1%**	**100.0%**

Figure 13: Construction of a twice-tiled factor index (Quality + Value Factors)
Source: FTSE Russell. (2017). *Multi-factor indexes: The power of tilting*

Double Sort Approach

The double sort procedure involves first constructing quantile portfolios where securities within the universe are being ranked according to the first factor characteristic. Securities within the first quartile are then reconstructed into quantile portfolios according to their respective scores relating to the second factor characteristic. The downside, however, would be a higher tracking error relative to a cap-weighted benchmark.

Part III: An Assessment of Factor Exposure

Once portfolios are being constructed, they could be assessed to identify their exposure to a specific factor of interest.

General Approach

The first approach to assessing a portfolio's factor exposure involves a summation of the portfolio's constituent security weights, each being multiplied by its respective Z-score as illustrated by the following (FTSE Russell, n.d.):

$$X = \sum_{j \in U} W_j * Z_j$$

where X represents the exposure of the portfolio to a factor of interest with W_j *and* Z_j *being the index weight and Z-score of a security, j respectively*

From here, one can derive the active factor exposure by taking the difference between the factor exposure of the target index and the factor exposure of the underlying index.

A returns based analysis can also be used to assess factor exposure. Excess returns of a portfolio are regressed against the returns of a single-factor portfolio to obtain the beta regression coefficients, which represent factor exposure.

Factor exposure is best determined using the first method as opposed to the second method when holdings based information is available as it is relatively unambiguous and allows an investor to derive the factor exposure at a specific point in time (FTSE Russell, n.d.). In contrast, the regression method presents multiple problems. One example would be the issue of selecting a time period that captures a variety of market conditions to eliminate bias.

Examples

In this section, we will provide a few examples of the active factor exposure of a few alternatively weighted indices. The capitalization weighted index is the underlying index.

(i) Factor indexes

The average active exposure of an index relative to a factor deviates from zero if the index exhibits the characteristics of the factor. Figure 14 clearly depicts this mathematical relation.

Index name	Average active exposure					
	Illiquidity	Residual momentum	Quality	Size	Value	(Low) volatility
Broad Illiquidity Factor Index	0.91	0.04	-0.03	0.96	0.01	-0.15
Residual Momentum Factor Index	0.01	0.44	0.00	0.00	-0.03	0.04
Broad Quality Factor Index	-0.04	0.00	0.50	-0.05	-0.02	0.12
Broad Size Factor Index	0.92	0.00	-0.02	1.20	0.03	-0.22
Broad Value Factor Index	0.01	-0.04	-0.04	0.02	0.39	0.04
Broad Volatility Factor Index	-0.08	0.03	0.08	-0.12	0.03	0.37

Figure 14: Average active exposure of different factor indexes
Source: FTSE Russell, *Factor exposures of smart beta indexes*

(ii) Equally weighted indexes

Figure 15 (below) provides the average active exposure (Sep 2001 to Jul 2015) of the equally weighted index to 5 different factors of interest—illiquidity, 12-month momentum, quality, size, value and volatility (low).

	Average active exposure					
	Illiquidity	12–month momentum	Quality	Size	Value	(Low) volatility
FTSE Developed Equally Weighted Index	1.21	-0.07	-0.01	1.42	0.08	-0.28

Figure 15: Average active exposure of an equally weighted index
Source: FTSE Russell, *Factor exposures of smart beta indexes*

In this example, the equally weighted index has a positive exposure to the illiquidity and size factors, and a negative exposure to the (low) volatility factor—which implies a volatility that is higher than the underlying index. The results of this analysis is largely expected because the equally weighted index overweights small capitalization stocks relative to the capitalization weighted index. Since stocks with a smaller market capitalization are typically less liquid and exhibit higher volatility, the average active exposure for these factors will deviate sizably from zero.

(i) Risk-based indexes

Here, we will provide examples of active factor exposure over time for some risk-based indexes.

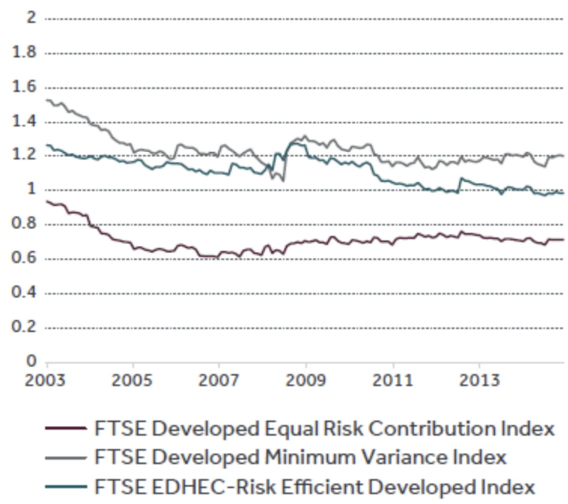


Figure 16: Active Size Exposure
Source: FTSE Russell, *Factor exposures of smart beta indexes*

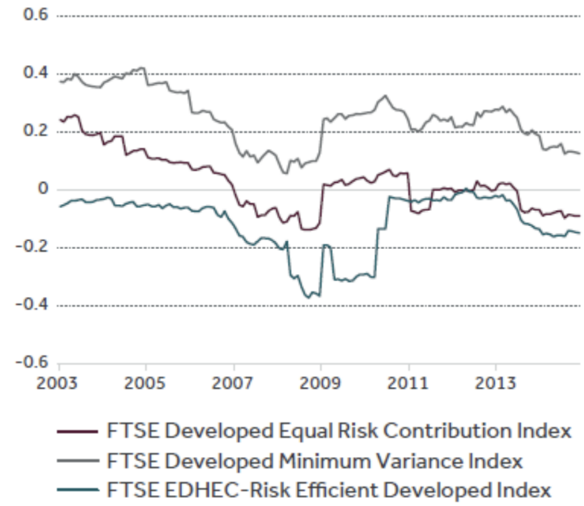


Figure 17: Active Volatility Exposure
Source: FTSE Russell, *Factor exposures of smart beta indexes*

	Average active exposure					
	Illiquidity	12–month momentum	Quality	Size	Value	(Low) volatility
FTSE Developed Equal Risk Contribution Index	0.63	-0.03	0.06	0.71	0.02	0.03
FTSE Developed Minimum Variance Index	1.03	0.00	0.12	1.24	0.04	0.23
FTSE EDHEC-Risk Efficient Developed Index	0.83	0.05	0.06	1.11	-0.01	-0.12

Figure 18: Average active exposure of FTSE Risk Indexes (Sep 2001 to Jul 2015)
Source: FTSE Russell, *Factor exposures of smart beta indexes*

Conclusion

In this paper, we have provided multiple examples of factor investing, which aim to improve portfolio performance by targeting specific drivers of return. These factors have been broadly categorized into macroeconomic drivers—which explain risk and return *across* asset classes—and style drivers—which explain risk and return *within* each asset class.

In practice, factor performance is largely cyclical. Portfolios allocated to *value* and *low-size* factors tend to outperform during times of economic expansion while portfolios allocated to *quality* and *min vol* factors exhibit superior performance during times of economic contraction. Some portfolio managers thus base their reallocation decisions on the existing economic outlook to maximize returns.

Factor investing remains a highly competitive field. Some areas of research include the application of factor investing methodology to fixed income assets, ESG investing and factor tilts, increased active management of factor exposure as well as improved ways of measuring factor exposure. Leading firms thus leverage on such proprietary research to optimize existing methodologies and improve outcomes for their clients.

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